
Si6000b Controlled Impedance Field Solver

User Guide

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Si6000b User Guide

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Personal Computer Requirements

Computer	IBM PC AT or compatible
Processor	Pentium 500 or better
Operating system	Microsoft Windows 95, Windows 98 Microsoft Windows NT, Windows 2000
System memory required	128MB recommended
Hard disk space required	100MB (min.)
Video standard	SVGA (1024 x 768 min.)
CDROM drive	
Mouse	Microsoft compatible
Spreadsheet	Microsoft Excel 97 or later

Guide To The Manual

Introduction	Introduces the Polar Instruments Si6000.
Introduction to controlled impedance PCBs	Brief discussion of the requirement for controlled impedance, impedance matching and calculation methods.
Transmission line structures	Brief discussion of the PCB structures used for controlled impedance.
Installation/set up	Hardware and software requirements and instructions for installing and uninstalling the software.
Using the Si6000 Quick Solver	Using the Si6000 Quick Solver for single impedance, propagation delay, inductance and capacitance calculations.
Using the Si6000 functions	Learning the Si6000 functions using the supplied sample Si6000 workbook. Using the Si6000 functions in your own applications in Si6000 expert mode.
Reference	Technical background to calculation of PCB track impedance. Discussion of field solving, Green's Functions and methods of calculation used in the Si6000.

Specifications

Supported structures

Single-ended

- Surface Microstrip
- Coated Microstrip
- Embedded Microstrip
- Symmetrical Stripline
- Offset Stripline
- Edge coupled Surface Microstrip
- Edge coupled Coated Microstrip
- Edge coupled Embedded Microstrip
- Edge coupled Symmetrical Stripline
- Edge coupled Offset Stripline
- Broadside Coupled Stripline
- Surface Coplanar Strips
- Surface Coplanar Waveguide
- Surface Coplanar Strips with Ground
- Surface Coplanar Waveguide with Ground
- Coated Coplanar Strips
- Coated Coplanar Waveguide
- Coated Coplanar Strips with Ground
- Coated Coplanar Waveguide with Ground
- Embedded Coplanar Strips
- Embedded Coplanar Waveguide
- Embedded Coplanar Strips with Ground
- Embedded Coplanar Waveguide with Ground

Differential

Offset Coplanar Strips

Offset Coplanar Waveguide

Differential Surface Coplanar Strips

Differential Surface Coplanar Waveguide

Differential Surface Coplanar Strips with Ground

Differential Surface Coplanar Waveguide with Ground

Differential Coated Coplanar Strips

Differential Coated Coplanar Waveguide

Differential Coated Coplanar Strips with Ground

Differential Coated Coplanar Waveguide with Ground

Differential Embedded Coplanar Strips

Differential Embedded Coplanar Waveguide

Differential Embedded Coplanar Strips with Ground

Differential Embedded Coplanar Waveguide with Ground

Differential Offset Coplanar Strips

Differential Offset Coplanar Waveguide

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Introduction to the Si6000

The Si6000 Controlled Impedance Field Solver

The Polar Si6000 Controlled Impedance Field Solver uses advanced field solving methods to calculate PCB trace impedance for most single-ended and differential circuit designs. Using Method of Moments, the Field Solver is able to provide rapid modelling for a wide range of microstrip, stripline and coplanar structures.

The Si6000 comprises the Si6000 Quick Solver, for rapid calculation of single PCB trace impedance values against significant PCB parameters and a comprehensive set of advanced field solving methods incorporated as user-defined functions in the popular Microsoft Excel spreadsheet to enable advanced function, e.g. sensitivity analysis.

Evaluating PCB structure behaviour

The Polar Si6000 runs within the Microsoft Windows environment. The Field Solver uses the familiar Microsoft Excel for Windows interface for easy graphing and data sharing. Using Excel's powerful Autofill and Chart Wizard features, the Si6000 can rapidly chart Z_0 against a varying parameter, providing easy comparison and evaluation of the behaviour of most popular controlled impedance structures.

Introduction to Controlled Impedance PCBs

Controlled impedance

The increase in processor clock speed and component switching speed on modern PCBs means that the interconnecting paths between components (i.e. PCB tracks) can no longer be regarded as simple conductors.

At fast switching speeds or high frequencies (i.e. for digital edge speeds faster than 1ns or analog frequencies greater than 300MHz) PCB tracks must be treated as *transmission lines*; i.e. for stable and predictable high speed operation the electrical characteristics of PCB traces and the dielectric of the PCB must be controlled.

One critical parameter is the *characteristic impedance* of the PCB track (the ratio of voltage to current of a wave moving down the signal transmission line); this will be a function of the physical dimensions of the track (e.g. track width and thickness) and the dielectric constant of the PCB substrate material and dielectric thickness. The impedance of a PCB track will be determined by its inductive and capacitive reactance, resistance and conductance. PCB impedances will typically range from 25Ω to 120Ω.

In practice a PCB transmission line typically consists of a line conductor trace, one or more reference planes and a dielectric material. The transmission line, i.e. the trace and planes, form the *controlled impedance*.

The PCB will frequently be multi-layer in fabrication and the controlled impedance can be constructed in several ways. However, whichever method is used the value of the impedance will be determined by its physical construction and electrical characteristics of the dielectric material:

- The width and thickness of the signal trace

- The height of the core or pre-preg material either side of the trace

- The configuration of trace and planes

- The dielectric constant of the core and pre-preg material

Impedance matching

Components themselves exhibit characteristic impedance so the impedance of the PCB tracks must be chosen to match the characteristic impedance of the logic family in use.

If the impedance of the PCB tracks does not match the device characteristic impedance multiple reflections will occur on the line before the device can settle. This can result in increased switching times or random errors in high speed digital systems. The value and tolerance of impedance will be specified by the circuit design engineer and the PCB designer, however, it will be left to the PCB manufacturer to conform to the designer's specification and verify the finished boards meet the specification.

Calculation methods

The SI6000 incorporates field solving for single-ended and differential impedance structures. The discrete numerical analysis in the SI6000 uses Boundary Finite Element Analysis to evaluate the residual field. A piecewise linear approximation is used with a weighted sub-division of the perimeter of the trace cross-section to predict the surface charge distribution on the trace. Knowing the boundary voltage conditions and the charge distribution allows evaluation of the Green's Function using Method of Moments integration to predict the capacitance of the structure. This in turn allows the impedance of the structure to be calculated.

Transmission Line Structures

Microstrip and Stripline Transmission Lines

Controlled impedance PCBs are usually produced using *microstrip and/or stripline transmission lines* in single-ended (unbalanced) or differential (balanced) configurations.

A micro strip line consists of controlled width conductive traces on a low-loss dielectric mounted on a conducting ground plane. The dielectric is usually made of glass-reinforced epoxy such as FR-4. For very high frequencies PTFE may be used. Other reinforcement/resin systems are also available.

For close spaced differences on woven glass reinforced dielectrics, refer to application note AP139 on the Polar Instruments web site, www.polarinstruments.com.

There are several configurations of PCB transmission line:

- Exposed, or surface, microstrip
- Coated microstrip (coating usually solder mask)
- Buried, or embedded, microstrip
- Centred stripline
- Dual (offset) stripline

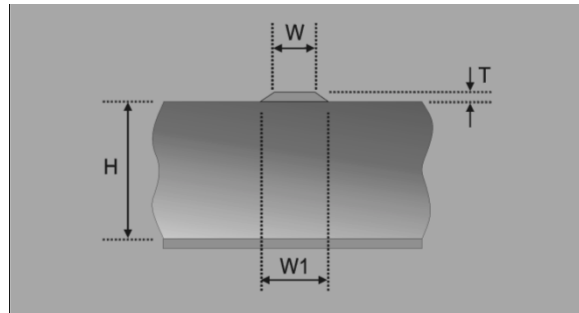
Single-ended Transmission Lines

Single-ended transmission lines are the commonest way to connect two devices (i.e. a single conductor connects the source of a device to the load of another device). The reference (ground) plane provides the return path.

Note that in the diagrams the trace is trapezoidal in profile and width, W , refers to the trace width nearest the upper surface, W_1 refers to the trace width nearest the lower surface.

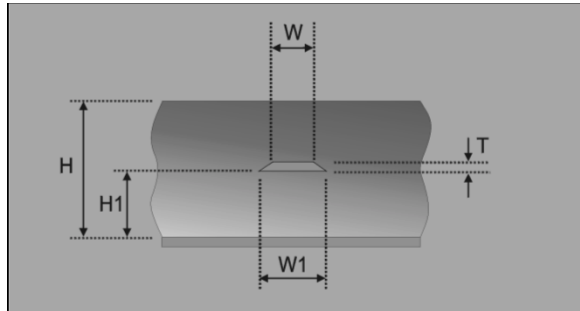
Surface Microstrip

In the diagram below (surface, or exposed, microstrip) the signal line is exposed (to air) and referenced to a power or ground plane.



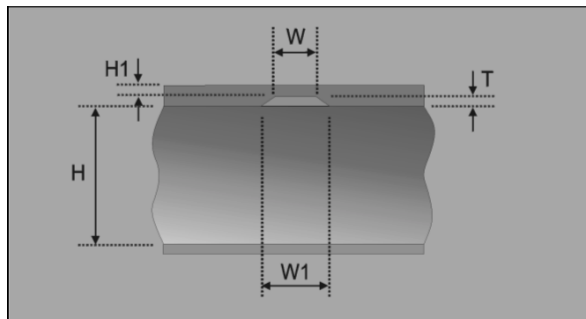
Embedded Microstrip

Embedded, or buried, microstrip is similar to the surface version, however the signal line is embedded in a dielectric and located a known distance from the reference plane.



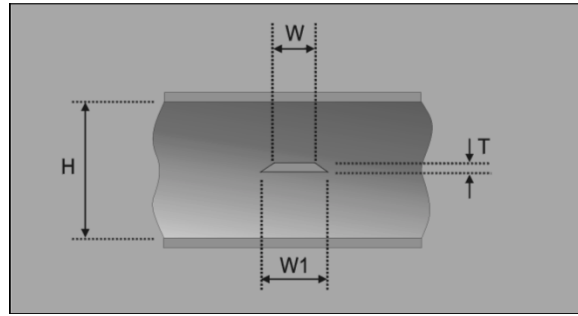
Embedding the signal line can lower the impedance by as much as 20% compared to an equivalent surface microstrip construction.

Coated Microstrip



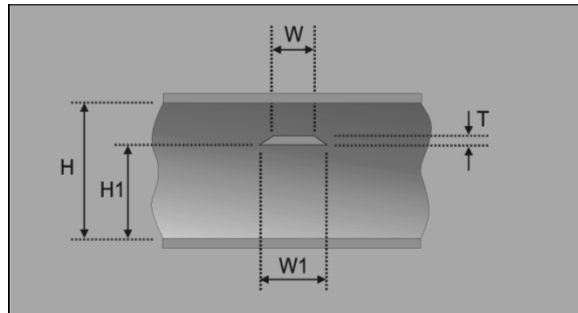
Coated microstrip is similar to the surface version, however the signal line is covered by a solder mask. This coating can lower the impedance by up to a few ohms depending on the type and thickness of the solder mask.

Symmetric Stripline



In this configuration the signal trace is sandwiched symmetrically between two planes. This is often difficult to achieve with normal manufacturing processes.

Offset Stripline



In this configuration the signal trace is sandwiched between two planes but is not equally spaced between the two planes. This construction is often referred to as Dual Stripline.

A second mirror trace will be positioned H_1 from the top ground plane. These two signal layers will be routed orthogonally (crossing at right angles so as to minimise the crossing area).

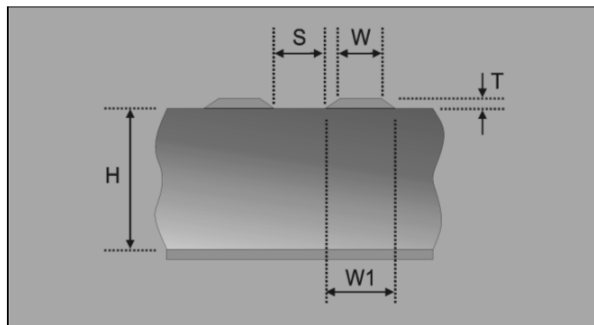
Differential Transmission Lines

The differential configuration (often referred to as a *balanced line*) is used when better noise immunity and improved timing are required. In differential mode the signal and its logical complement are applied to the load.

The balanced line thus has *two* signal conductors and an associated reference plane or planes as in the equivalent single-ended (unbalanced) case. Fields generated in the balanced line will tend to cancel each other, so EMI and RFI will be lower than with the unbalanced line. External noise will be "common-moded out" as it will be equally sensed by both signal lines.

Note that in the following diagrams (except the Broadside-coupled Stripline) the traces are trapezoidal in profile and width, W , refers to the trace width nearest the upper surface, W_1 refers to the trace width nearest the lower surface.

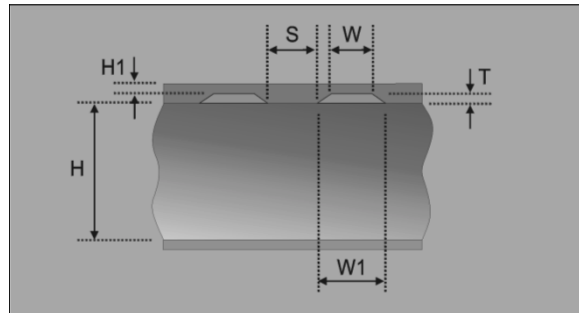
Edge-coupled Surface Microstrip



In this construction the gap between the traces defines the coupling factor and hence the differential impedance. The etch factor, plating density and undercut will make this construction

simple to manufacture, but with a wider tolerance due to the extra processing required on external layers.

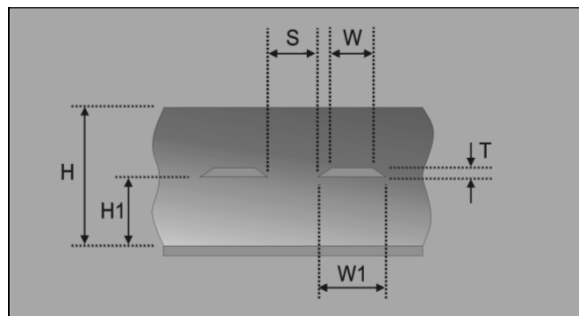
Edge-coupled Coated Microstrip



As in the case of the Surface Microstrip this construction is simple to fabricate, but the extra process of adding solder mask coating can cause impedance variations.

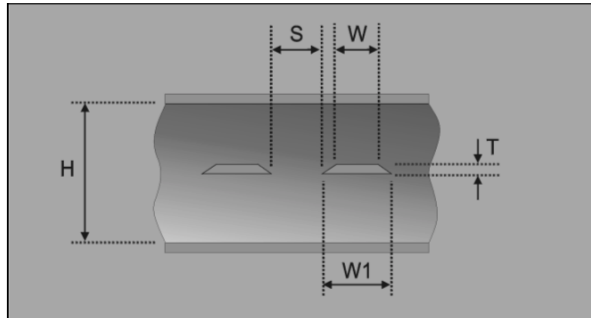
This construction is particularly sensitive to solder mask flooding with LPI (Liquid Photo Imagable) solder mask. This causes the dielectric constant in the edge coupling region to vary, depending on flood depth.

Edge-coupled Embedded Microstrip



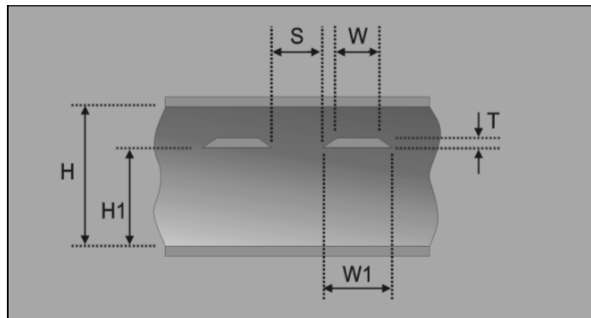
The reduced processing of internal layers makes the Edge-coupled Embedded Microstrip construction easy to fabricate with more consistent results than the equivalent surface trace structure.

Edge-coupled Symmetric Stripline



The Edge-coupled Symmetric Stripline is the simplest differential construction possible, though it may prove difficult to maintain the signal traces exactly in the centred position. Any offset from the centred position will reduce the impedance.

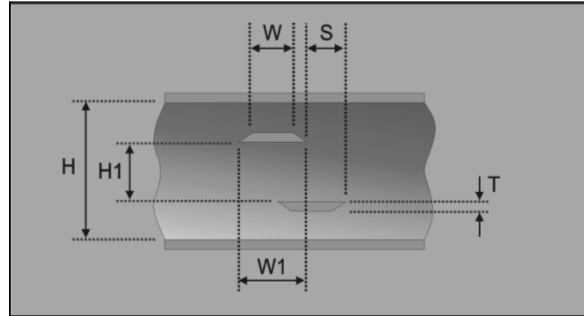
Edge-coupled Offset Stripline



As in the case of the single-ended Offset Stripline construction this structure can be made up as a dual construction with a mirrored edge-coupled

differential pair set a distance H_1 from the upper reference plane. The lower pair are routed orthogonally to minimise layer to layer coupling and crosstalk.

Broadside-coupled Stripline



This apparently simple construction is actually one of the most difficult to fabricate to produce consistent impedance results.

Despite having internal layers with minimal processing, the most common structure is that with both traces overlaid for maximum coupling.

Inner-layer mis-registration and slight offsets and differences in etching combine to make this more difficult to achieve consistent results, particularly if the traces are fine-line.

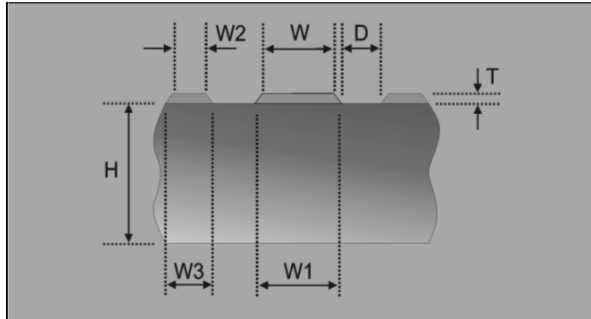
Note that in the Broadside-coupled Stripline case the traces are trapezoidal in profile and width, W , refers to the trace width nearest the surfaces, W_1 refers to the trace width nearest the Center Line.

Coplanar Lines

Most Microstrip and Stripline transmission line structures can be manufactured in a Coplanar version. Coplanar structures show only minor dispersion effects compared to microstrip lines.

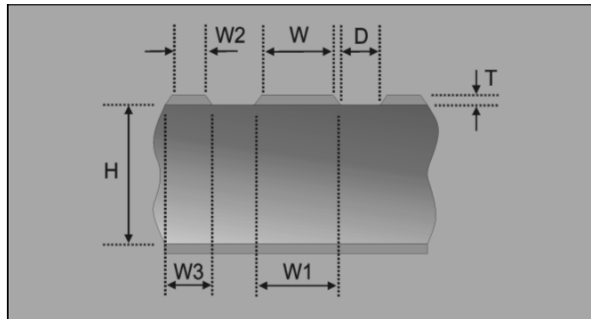
Coplanar Lines incorporate ground conductors adjacent to the controlled impedance trace(s) in the same plane as the trace(s).

Surface Coplanar Strips



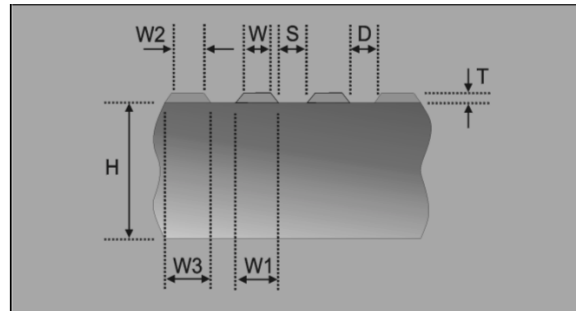
Surface Coplanar Strips with Ground

Coplanar Lines may be constructed with or without a ground plane underneath the controlled impedance trace(s).



This structure is an example of a controlled impedance trace on a single sided board which will typically be used in consumer applications.

Differential Surface Coplanar Strips



Installing the Si6000 Field Solver

Activating the Si6000

Minimum system requirements

The Si6000 requires a system with the following characteristics:

- IBM PC compatible
- Pentium (500MHz or better recommended)
- Mouse
- Parallel port
- 100MB free hard disk space
- 128MB RAM recommended
- Windows 95/98/ME, Windows NT 4.0/2000
- Excel 97, 2000 or 2002

Installing the software key

It will be necessary to activate the Si6000 software prior to use. Insert the software key into the parallel printer port socket; if necessary, plug the printer cable into the software key.

Note: If installing the software key on a Microsoft Windows NT or 2000 system it will be necessary to be logged on as Administrator.

Uninstalling the software

To uninstall the Si6000 software click the Windows Start button and choose Settings and Control Panel.

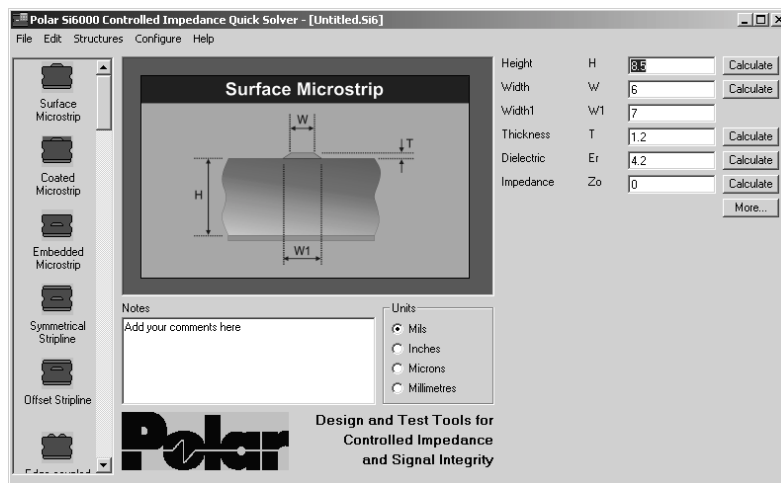
Double-click Add/Remove Programs and choose Si6000b from the list. Click Remove.

Using the Si6000 Quick Solver

The Si6000 Quick Solver

The Si6000 Quick Solver allows the operator to perform rapid single calculations of PCB trace values against significant PCB parameters. The Si6000 Quick Solver solves for impedance, propagation delay and inductance and capacitance per unit trace length.

Click the Si6000 Quick Solver icon on the desktop to start the Si6000 Quick Solver.



Calculating impedance

Click on the configuration from the Structures menu or from the Structures bar.

Select the dimension units (mils, inches, microns or millimetres) from the Units option group.

Enter the values for:

H (Height) — dielectric thickness

W and W1 (Width) — signal trace width
(allowing for finished etch factor)

T (Thickness) — signal trace thickness

Er — dielectric constant

into the text boxes and press the Calculate button

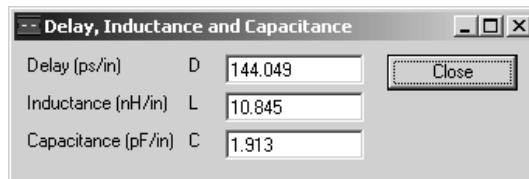
The calculated impedance will appear in the Impedance (Z_0) box.

Add explanatory notes on your particular construction, if necessary, in the Notes text box.

Calculating propagation delay, inductance and capacitance

Click on the configuration from the Structures menu or from the Structures bar.

Enter the parameter values as described above into the text boxes and press the More... button.



Delay, Inductance and Capacitance		
Delay (ps/in)	D	144.049
Inductance (nH/in)	L	10.845
Capacitance (pF/in)	C	1.913

The Si6000 displays the results of propagation delay, inductance and capacitance in the selected units. Press Close to exit.

Quick solving for board parameters

The Quick Solver can quickly solve for board parameters given a nominal impedance value.

Height	H	8.5	Calculate
Width	W	6	Calculate
Width1	W1	7	
Thickness	T	1.2	Calculate
Dielectric	Er	4.2	Calculate
Impedance	Zo	0	Calculate
			More...

Enter the given board dimensions in their associated fields and the nominal impedance value in the Impedance field and click the Calculate button against the unknown dimension, e.g. Height, Width, etc.

Setting Si6000 parameter limits

The Si6000 is designed to work with “real world” values. If the parameter values used in calculation are beyond the Si6000 limits, the Si6000 returns a value of zero.

The user is able to control the values used by the Si6000 field solving engine during calculation.

Click the Configure menu; the Si6000 Configuration dialog is displayed.

Enter the values for minimum and maximum for each parameter and number of calculation iterations (Si6000 Quick Solver Tries).

Saving and recalling results

Impedance calculation results for a board type or vendor, for example, may be saved to disk and recalled for future reference.

From the File menu choose the Save As... command. Choose a name and destination and press Save.

The program will only save calculated results.

To recall a set of results choose Open... from the File menu and choose the desired results file and press Open.

Printing results

Choose Print from the File menu to print a hard copy of the Si6000 Quick Solver screen.

Copying Quick Solver parameters to Excel

Selecting this command from the Edit menu takes the current parameters from the Quick Solver and places them on the Clipboard in Microsoft Excel format.

The parameter values may then be pasted to a suitable location in an Si6000 or Si6000Expert workbook. The values are placed horizontally beginning at the active cell location. The number of cells required depends upon the structure chosen. Ensure no important data are overwritten in the process.

The operator can now use Excel's Paste Function command to insert the associated Si6000 function using the pasted Quick Solver parameter values as arguments.

(See *Using the Si6000 Field Solver*.)

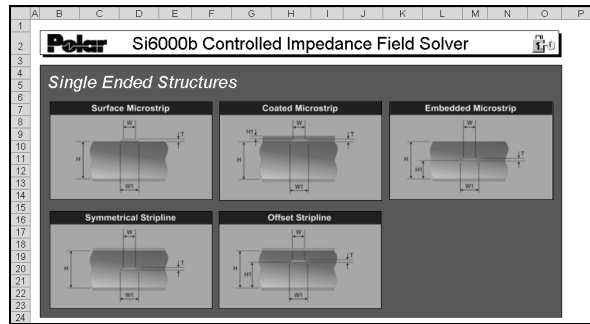
Using the Si6000 Field Solver

Starting the Field Solver

Note: the graphics displayed in this section are based on Microsoft Excel 2002. Dialog box graphics from earlier versions of Microsoft Excel may display slight differences from those shown here.

The Field Solver functions for each controlled impedance structure are built into Microsoft Excel workbooks Si6000b.xls and Si6000Expert.xls as user-defined functions. This allows rapid and convenient analysis of board trace characteristics such as impedance, propagation delay, inductance and capacitance against several varying board parameters. In addition to the Field Solver functions, the Si6000b.xls workbook includes pre-built sample data worksheets incorporating tables of functions and their associated parameters.

Double click the Field Solver icon on the desktop; Microsoft Excel opens the Si6000b.xls workbook at the Si6000 index sheet.



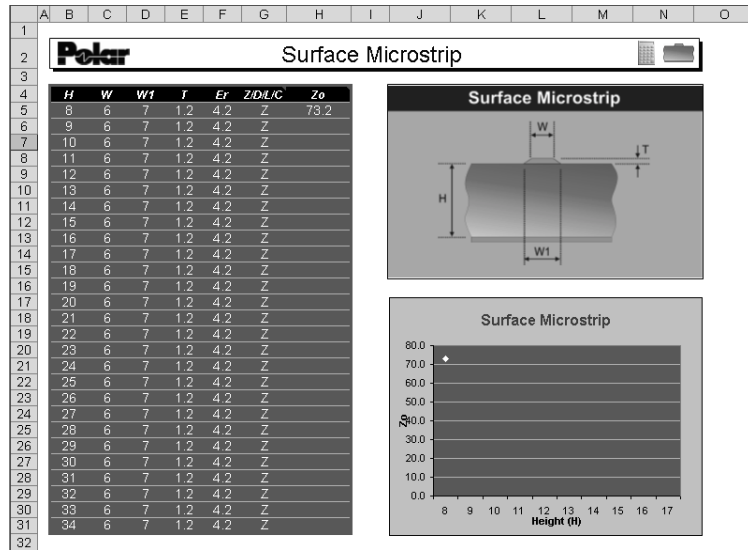
Si6000 index sheet — Single Ended Structures

Si6000 structure categories

The index sheet displays the Si6000 structure categories:

- Single Ended
- Differential
- Surface Coplanar
- Coated Coplanar
- Embedded Coplanar
- Offset Coplanar
- Differential Surface Coplanar
- Differential Coated Coplanar
- Differential Embedded Coplanar
- Differential Offset Coplanar

To select a structure scroll to the category and click on its graphic. e.g. Surface Microstrip. The Si6000 selects the associated worksheet.



Surface Microstrip sample worksheet

Moving through the structure sheets

Structure sheets may also be selected via the Tab Scrolling Buttons, .

Click the buttons to select the first, previous, next or last structure sheets.

Alternatively, use the Ctrl + Page Up/Ctrl + Page down keys to move to the previous/next sheet.

To move directly to a structure, right click the Tab Scrolling Buttons to display the list of structure sheets.



Select the structure from the list. If necessary choose **More Sheets...** to display the complete list of structures.

Calculating trace characteristics

Each worksheet includes a pre-built sample application, incorporating a table of typical dimensions for use with the function associated with the structure and a chart displaying the change in impedance (Z_0), propagation delay (D), inductance (L) or capacitance (C) against structure dimensions (Z_0 is shown against a varying Height (H) in the sample chart).

	H	W	W1	T	Er	Z/D/L/C	Z ₀
4							
5	8	6	7	1.2	4.2	Z	73.2
6	9	6	7	1.2	4.2	Z	
7	10	6	7	1.2	4.2	Z	
8	11	6	7	1.2	4.2	Z	
9	12	6	7	1.2	4.2	Z	
10	13	6	7	1.2	4.2	Z	
11	14	6	7	1.2	4.2	Z	
12	15	6	7	1.2	4.2	Z	
13	16	6	7	1.2	4.2	Z	
14	17	6	7	1.2	4.2	Z	
15	18	6	7	1.2	4.2	Z	
16	19	6	7	1.2	4.2	Z	

Sample table with increasing values of H

The sheet opens with the single value of Z_0 calculated for the structure dimensions shown in row 5. The function is located in the cell labelled Z_0 , the parameters for the function are contained in the associated cells labelled H, W, W1, etc.

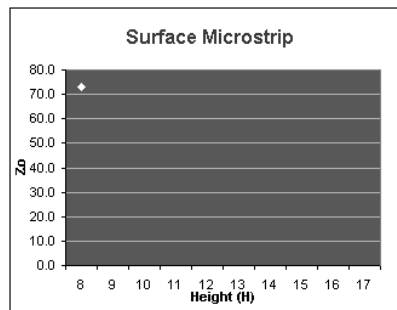
Choosing the calculation type, Z, D, L, C

To calculate other characteristics for the selected parameters, enter the value D, L or C in the associated cell in the Calc Type column (labelled **Z/D/L/C**), move to another cell and press the Calculate button. Re-label the results column if necessary.

<i>H</i>	<i>W</i>	<i>W1</i>	<i>T</i>	<i>Er</i>	<i>Z/D/L/C</i>	<i>D</i>
8	6	7	1.2	4.2	D	5680.4

Charting against varying board parameters

The Si6000 sheet opens with the value of Z_0 against H for the structure dimensions shown in row 5 and charted as shown below.



To chart the change in Z_0 , D , L , or C as H changes over a range of values, use the Excel Fill Handle to copy the function formula down into the associated cells.

(To activate the Fill Handle, move the mouse to the lower right corner of the active cell. The mouse changes to a black plus sign.

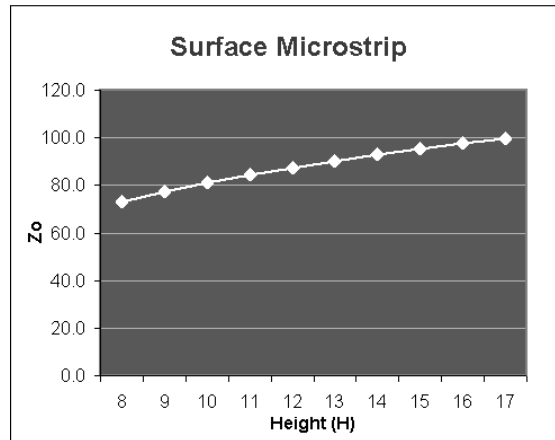
If the Fill Handle does not appear, select **Tools|Options|Edit** and tick the **Allow cell drag and drop** check box.)

<i>Er</i>	<i>Z/D/L/C</i>	<i>D</i>
4.2	Z	73.2
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	
4.2	Z	

Use Excel's Fill Handle to copy the formula down

Press the Calculate button to recalculate the worksheet. (The Si6000b Workbook sets Excel's Calculation mode to Manual; see **Tools|Options|Calculation**). The Si6000 field solves for all associated rows and refreshes the chart.

<i>H</i>	<i>W</i>	<i>W1</i>	<i>T</i>	<i>Er</i>	<i>Z/D/L/C</i>	<i>D</i>
8	6	7	1.2	4.2	Z	73.2
9	6	7	1.2	4.2	Z	77.3
10	6	7	1.2	4.2	Z	80.9
11	6	7	1.2	4.2	Z	84.2
12	6	7	1.2	4.2	Z	87.3
13	6	7	1.2	4.2	Z	90.1
14	6	7	1.2	4.2	Z	92.7
15	6	7	1.2	4.2	Z	95.2
16	6	7	1.2	4.2	Z	97.5
17	6	7	1.2	4.2	Z	99.7
18	6	7	1.2	4.2	Z	101.8
19	6	7	1.2	4.2	Z	103.7



Plot of Z_0 as Height (H) varies

Choosing other parameters

Z_0 , D, L and C can be plotted against any of the function parameters.

For example, to display Z_0 as E_r varies, in the example reset H to a single value, e.g. 8, and plot Z_0 against changes of E_r between in 0.1 increments.

Changing the parameters

Select the first value in the Height column and use the Fill Handle to fill down to row 16 with the value 8.

Change the first value in the E_r column to 3.8, change the second value to 3.85 then select *both* cells.

Use the Fill Handle to fill down to row 16; Excel detects the two cell values as an incrementing

sequence and fills the column accordingly with values increasing at 0.05 intervals.

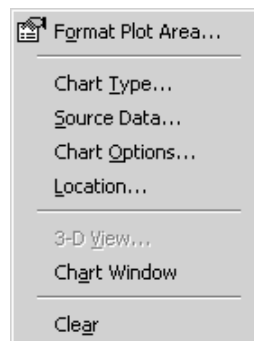
Click the Calculate icon to refresh the Z_0 column.

<i>H</i>	<i>W</i>	<i>W1</i>	<i>T</i>	<i>Er</i>	<i>Z/D/L/C</i>	<i>Z</i>
8	6	7	1.2	3.8	Z	76.4
8	6	7	1.2	3.85	Z	75.9
8	6	7	1.2	3.9	Z	75.5
8	6	7	1.2	3.95	Z	75.1
8	6	7	1.2	4	Z	74.7
8	6	7	1.2	4.05	Z	74.4
8	6	7	1.2	4.1	Z	74.0
8	6	7	1.2	4.15	Z	73.6
8	6	7	1.2	4.2	Z	73.2
8	6	7	1.2	4.25	Z	72.9
8	6	7	1.2	4.3	Z	72.5
8	6	7	1.2	4.35	Z	72.1

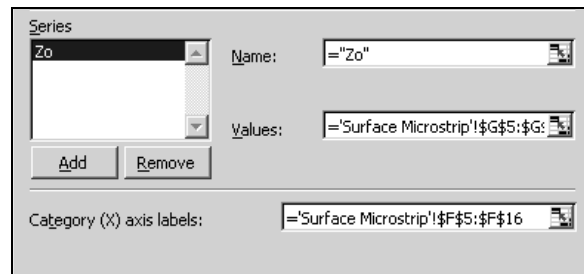
Modifying the chart

It will be necessary to modify the chart to reflect the new scales and Category axis.

Right click the chart area and choose Source Data...



From the Source Data dialog box, click the Series tab; the Series page shows the source data cell ranges for the chart.



Series

Zo

Name: =Zo

Values: =Surface Microstrip!\$G\$5:\$G\$16

Add Remove

Category (X) axis labels: =Surface Microstrip!\$F\$5:\$F\$16

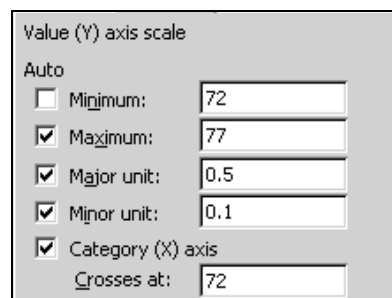
Click the Collapse Dialog button, (), in the Category (X) axis labels: edit box and select the range of Er values charted.

Click the button again to restore the dialog box and press OK.

Click the Category Axis Title label and replace the H with Er.

Right click the value axis and choose Format Axis...

Choose the Scale tab and change the values as necessary for Minimum and Maximum scale values.



Value (Y) axis scale

Auto

☐ Minimum: 72

☒ Maximum: 77

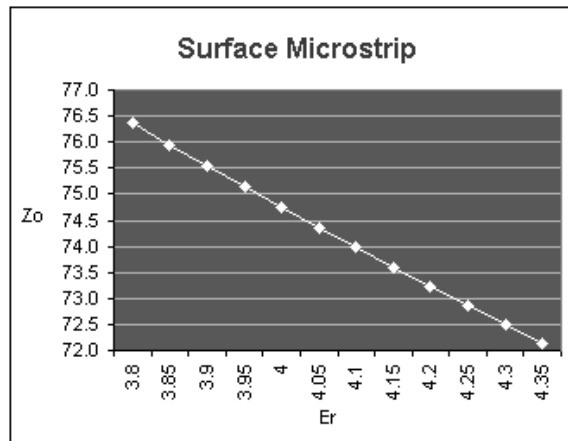
☒ Major unit: 0.5

☒ Minor unit: 0.1

☒ Category (X) axis

Crosses at: 72

Press OK. The chart should appear as shown below.



Repeat the procedure for other parameter values.

Using the Si6000 functions in other workbooks

The Si6000b functions supplied by the Si6000 workbooks, Si6000b.xls or Si6000bExpert.xls, are available for use in other workbooks.

Prior to using any of the Si6000b functions it will be necessary to ensure the Si6000b.xls workbook or Si6000bExpert.xls is open. In this discussion the worksheet is assumed to refer to the Si6000b.xls workbook.

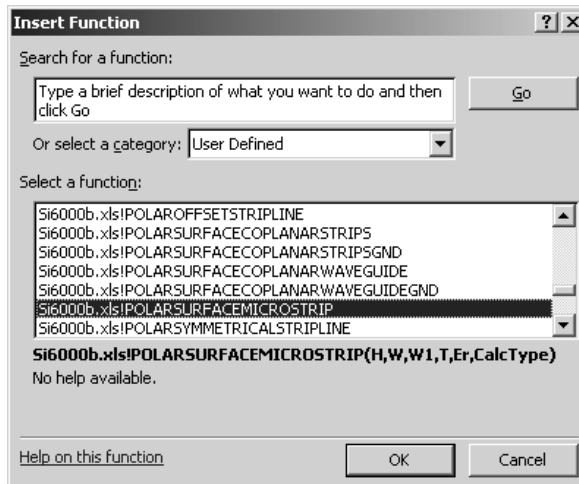
The Si6000 functions use the board parameters, H, W, E_r etc. as arguments. Parameter values can be derived from existing data in worksheet cells or inserted into the Formula Palette directly.

Begin a new worksheet.

In the example below the values for a Surface Microstrip structure are contained in cells B4 to G4. We insert the Si6000 Surface Microstrip function into cell H4.

	A	B	C	D	E	F	G	H	I
1									
2									
3		H	W	W1	T	Er	Calc Type	Zo	
4		8	6	7	1.2	3.8	Z		
5									
6									

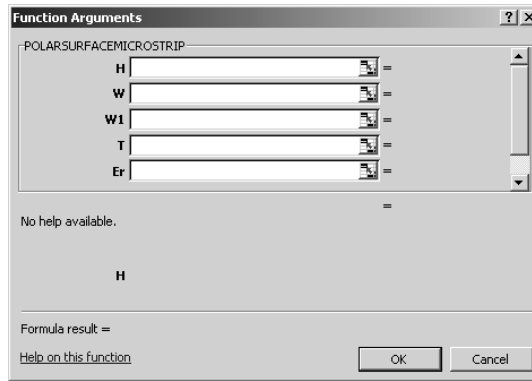
To use a function, from the **Insert** menu choose **Function...** or click the **Insert Function** button  on the tool bar. The Paste Function dialog box is displayed. Choose the **User Defined** category to display the user-defined functions.



Scroll to the Si6000 functions; click the function associated with the surface microstrip structure (Si6000b.xls!POLARSURFACEMICROSTRIP() in this example) and click OK: the Formula Palette is displayed.

Using the Formula Palette to entering formulas

The Excel Formula Palette can help reduce errors when entering function parameters. The Formula Palette creates an edit box for each argument in the function.



Click the Collapse Dialog button () in the **H** edit box and select cell B4: click the button again. Tab through the other edit boxes and repeat the procedure for each value.

As the function is entered, the Formula Palette displays the value of each of its arguments, the current result of the function, and the current result of the entire formula. When the last value is entered Excel calculates and displays the final result.

Function Arguments

POLARSURFACEMICROSTRIP

W C4 = 6

W1 D4 = 7

T E4 = 1.2

Er F4 = 3.8

CalcType G4 = "Z"

= 76.36252217

No help available.

CalcType

Formula result = 76.36252217

[Help on this function](#)

OK Cancel

Press OK to close the Formula Palette and complete the formula.

	A	B	C	D	E	F	G	H	I
1									
2									
3		H	W	W1	T	Er	Calc Type	Zo	
4		8	6	7	1.2	3.8	Z	76.36252	
5									
6									

Z_0 calculated for a single set of values

To calculate Z_0 over a range of parameter values, select the data and formula (cells B4 to H4) and use the Fill Handle to copy down as necessary.

Select each column of cells as appropriate and enter the new parameter values.

Hint: to fill a range of cells with a single value select the range, type the value and press Shift + Enter.

Press Shift + F9 to recalculate the sheet.

If necessary use the Increase decimal/Decrease Decimal buttons   to select the required number of decimal places.

Format as required.

	A	B	C	D	E	F	G	H	I
1									
2									
3		H	W	W1	T	Er	Calc Type	Zo	
4		8	6	7	1.2	3.80	Z	76.36	
5		8	6	7	1.2	3.85	Z	75.95	
6		8	6	7	1.2	3.90	Z	75.54	
7		8	6	7	1.2	3.95	Z	75.14	
8		8	6	7	1.2	4.00	Z	74.74	
9		8	6	7	1.2	4.05	Z	74.35	
10		8	6	7	1.2	4.10	Z	73.97	
11		8	6	7	1.2	4.15	Z	73.59	
12		8	6	7	1.2	4.20	Z	73.22	
13		8	6	7	1.2	4.25	Z	72.86	
14		8	6	7	1.2	4.30	Z	72.50	
15									

Z₀ calculated for changing E_r

Charting results

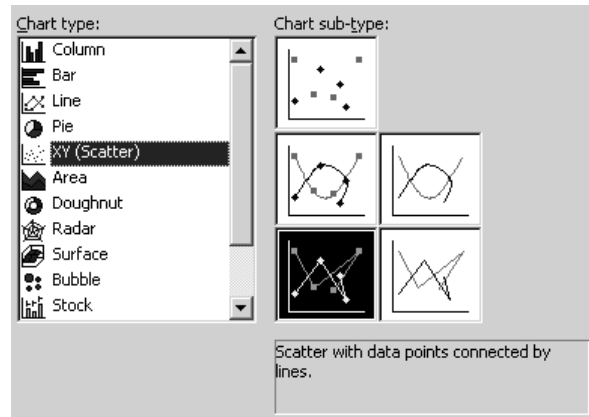
Use the Excel Chart Wizard to chart the results.

Select the area to be charted: in this example the E_r and Z_0 ranges (to select non-adjacent ranges, press Ctrl while dragging the mouse over each range).

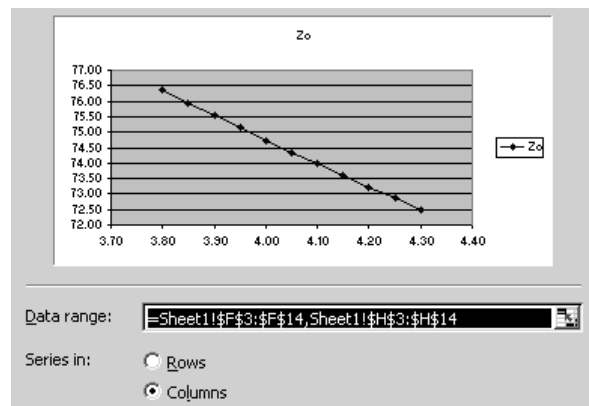
E	F	G	H	I
T	E_r	Calc Type	Z_0	
1.2	3.80	Z	76.36	
1.2	3.85	Z	75.95	
1.2	3.90	Z	75.54	
1.2	3.95	Z	75.14	
1.2	4.00	Z	74.74	
1.2	4.05	Z	74.35	
1.2	4.10	Z	73.97	
1.2	4.15	Z	73.59	
1.2	4.20	Z	73.22	
1.2	4.25	Z	72.86	
1.2	4.30	Z	72.50	

Click the Chart Wizard button  to start the Chart Wizard.

From Step 1, Chart Type, of the Chart Wizard, choose Chart Type: XY (Scatter) type and Chart sub-type: Data points connected by lines

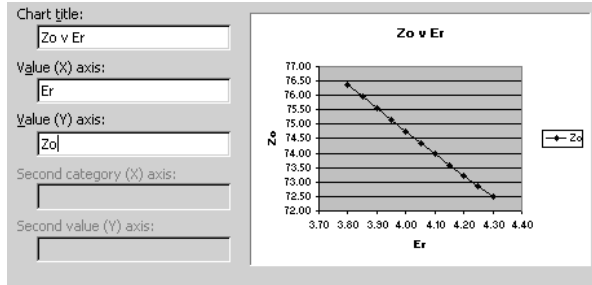


Click Next to select Step 2, Chart Source Data.

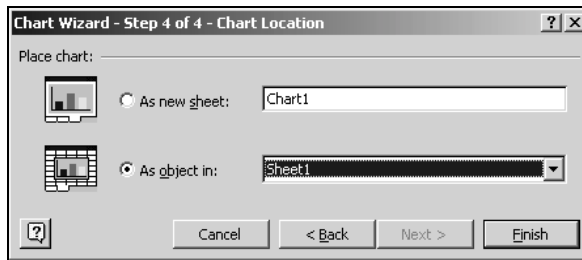


Check the Data Range: cell references are correct: leave the Series in: option set to Columns.

Click Next to select Step 3, Chart Options, and add titles for the chart and its axes and (optionally) remove the legend.



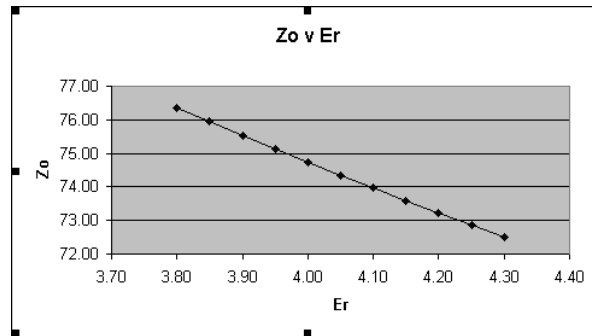
Click Next to select Step 4, Chart Location.



Use this dialog box to choose where Excel creates the chart.

To place the chart on a new chart sheet, click **As new sheet:**, and type a name for the new chart sheet.

To embed the chart on the worksheet, click **As object in:**, select a sheet name from the As object in box, and click Finish.



Choose the **As object in:** option then drag and size the embedded chart as required on the worksheet.

To modify the data series (e.g. line weight, marker style etc.) right click the chart line and choose Format Data Series...

Line

- ☒ Automatic
- ☐ None
- ☐ Custom
- Style:
- Color: Automatic
- Weight:
- ☐ Smoothed line
- Sample:

Marker

- ☐ Automatic
- ☐ None
- ☒ Custom
- Style: ◆
- Foreground: Automatic
- Background: Automatic
- Size: 3 pts
- ☐ Shadow

Change the series format as required.

Using the Si6000 to plot multiple data series

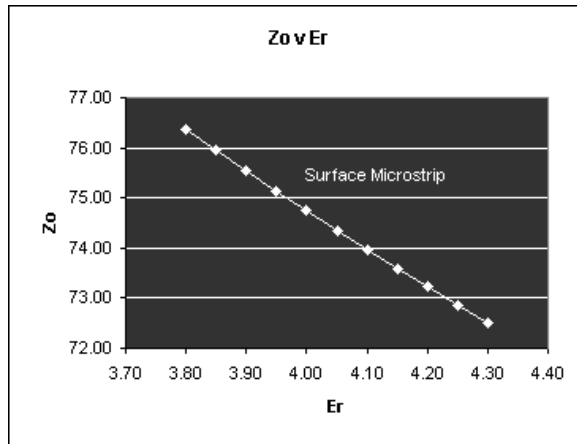
Inserting the first data series

Supply the data and plot the data series for Surface Microstrip as described earlier.

Surface Microstrip							
H	W	W1	T	Er	Calc Type	Zo	
8	6	7	1.2	3.80	Z	76.36	
8	6	7	1.2	3.85	Z	75.95	
8	6	7	1.2	3.90	Z	75.54	
8	6	7	1.2	3.95	Z	75.14	
8	6	7	1.2	4.00	Z	74.74	
8	6	7	1.2	4.05	Z	74.35	
8	6	7	1.2	4.10	Z	73.97	
8	6	7	1.2	4.15	Z	73.59	
8	6	7	1.2	4.20	Z	73.22	
8	6	7	1.2	4.25	Z	72.86	
8	6	7	1.2	4.30	Z	72.50	

In this example, plot Z_0 against E_r .

To add comments, titles, etc., to the chart, click into the Excel Formula Bar, type the text and press <Return>; place the text as required.



To format any chart item, right click the item and change its properties via the short cut menu.

Adding the second data series

Supply the data for the Coated Microstrip structure.

Coated Microstrip							
H	H1	W	W1	T	Er	Calc Type	Zo
8	1	6	7	1.2	3.80	Z	70.72
8	1	6	7	1.2	3.85	Z	70.30
8	1	6	7	1.2	3.90	Z	69.89
8	1	6	7	1.2	3.95	Z	69.48
8	1	6	7	1.2	4.00	Z	69.08
8	1	6	7	1.2	4.05	Z	68.69
8	1	6	7	1.2	4.10	Z	68.31
8	1	6	7	1.2	4.15	Z	67.93
8	1	6	7	1.2	4.20	Z	67.56
8	1	6	7	1.2	4.25	Z	67.13
8	1	6	7	1.2	4.30	Z	66.74

Right click the chart and choose Source Data... from the menu. Click the Series tab.

Series

Zo

Name:

=Sheet1!\$G\$4

X Values:

=Sheet1!\$F\$5:\$F\$13

Y Values:

=Sheet1!\$G\$5:\$G\$13

Add

Remove

Click the Add button to add a second series to the chart; in this case we use the same values for the X axis. Click the Collapse Dialog button and select the values in the E_r column.

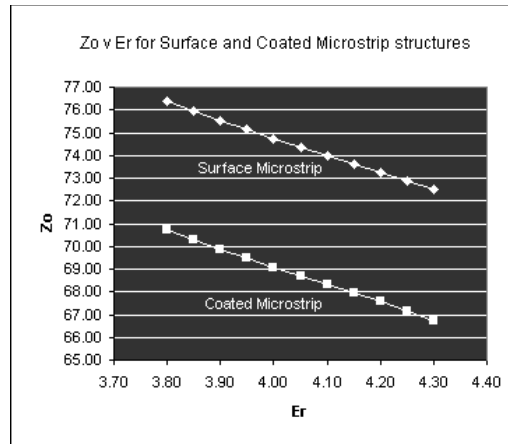
T	Er	Calc Type
1.2	3.80	Z
1.2	3.85	Z
1.2	3.90	Z
1.2	3.95	Z
1.2	4.00	Z
1.2	4.05	Z
1.2	4.10	Z
1.2	4.15	Z
1.2	4.20	Z
1.2	4.25	Z
1.2	4.30	Z

Repeat for the Y axis; select the values in the Coated Microstrip Z_0 column.

Er	Calc Type	Zo
3.80	Z	70.72
3.85	Z	70.30
3.90	Z	69.89
3.95	Z	69.48
4.00	Z	69.08
4.05	Z	68.69
4.10	Z	68.31
4.15	Z	67.93
4.20	Z	67.56
4.25	Z	67.13
4.30	Z	66.74

Add the name Coated Microstrip to the Name text box, click the Z_0 series from the Series list and add the name Surface Microstrip; press OK.

Right click the Y axis and choose Format Axis..., choose the Scale tab and specify 65 as the minimum value.



Use the Formula Bar to add explanatory text as necessary.

Using more complex models

Calculating the effect of etch back

In this example we consider the effect of PCB trace side wall slope. As part of the process we chart the change in impedance due to variations in dielectric thickness (H) and trace width (W). We use the surface microstrip structure.

Begin by entering the parameter values for the surface microstrip structure in cells A2:F2.

	A	B	C	D	E	F	G
1	H	W	W1	T	Er	Z/D/L/C	Zo
2	10.00	6.70	7.00	0.60	4.20	Z	82.3
3							
4							
5		=C2-D7					
6							
7							
8							
9							
10							
11							
12							

The etch back factor will be a variable so assign W1 a value of **7.00**, locate the etch back factor in cell D7 and define W as C2–D7 (i.e. W1 minus etch back factor). Assign a value of **0.3** as etch back factor and insert the Surface Microstrip function into cell F6;

=Si6000b.xls!POLARSURFACEMICROSTRIP(A2,B2,C2,D2,E2,F2)

Press Shift-F9 to calculate.

Calculating the effect of variations in H

Next, chart the effect of varying height H in 0.05 steps. Create references to cells A2–F7 in cells A22–F22 and change each reference B22–F22 to mixed.

= Si6000b.xls!POLARSURFACEMICROSTRIP(A22,B\$22,C\$22,D\$22,E\$22,F\$22)

14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							
28							
29							
30							

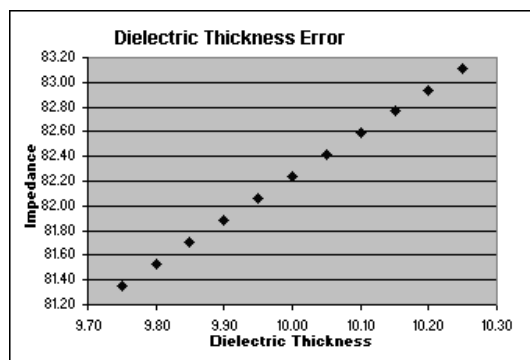
Copy the formula in G22 up to cell G17 and down to G27 as shown above

Create a step value of **0.05** in D14, enter the equation **=A22+\$D\$14** in cell A21 and fill it up to A17.

Enter the equation **=A22-\$D\$14** in cell A23 and fill it down to A27.

Use the Auditing Toolbar Trace Precedent and Dependent arrows to check references are as shown above. Press Shift-F9 to recalculate.

Select ranges H(var) A17:A27 and Z₀(H) G17:G27 and chart; the chart should appear as below.



Charting trace width error

Next we chart the effect of varying the trace width with a fixed trace side slope

33			Step	0.10	
34					
35		W(var)	W1(W/var)		Zo(W)
36		6.20	6.60		84.76
37	=C40-\$D\$7	6.30	6.60	=C41-\$D\$33	84.26
38		6.40	6.70		83.76
39		6.50	6.80		83.26
40		6.60	6.90		82.78
41	10.00	6.70	7.00	0.60	82.30
42		6.80	7.10		81.83
43		6.90	7.20		81.37
44		7.00	7.30		80.91
45	=C42-\$D\$7	7.10	7.40	=C41+\$D\$33	80.46
46		7.20	7.50		80.01
47					
48					
49	=Si6000b.xls!POLARSURFACEMICROSTRIP(A\$41,B41,C41,D\$41,E\$41,F\$41)				

Create references to A2:F2 in cells A41:F41.

Enter the formula

=Si6000b.xls!POLARSURFACEMICROSTRIP(A\$41,B41,C41,D\$41,E\$41,F\$41)

in cell G41 and copy it up to G36 and down to G46 as shown. (Note the relative references to B41 and C41.)

Create a step value of 0.10 in cell D33.

Enter formula **=C40-\$D\$7** in cell B40 and fill up to B36.

Enter formula **=C41-\$D\$33** in cell C40 and fill up to C36.

Enter formula **=C42-\$D\$7** in cell B42 and fill down to B46.

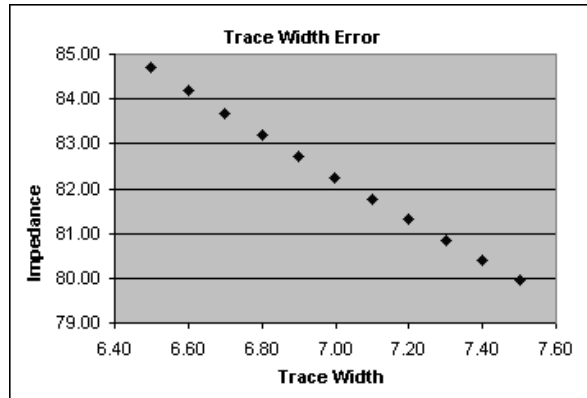
Enter formula **=C41+\$D\$33** in cell C42 and fill down to C46.

Use the auditing arrows to check cell precedents and dependencies.

Recalculate.

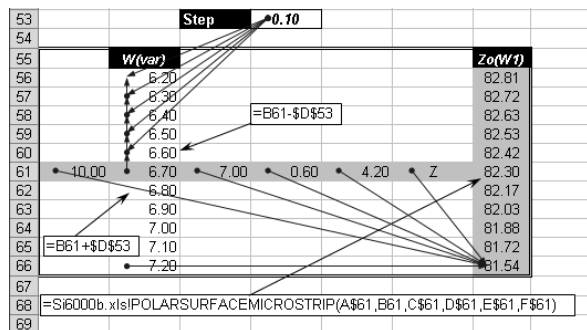
Select ranges C36:C46 and G36:G46 and chart.

The trace width error chart should appear as shown below.



Charting etch back error

Finally, we chart the effect of etch back error.



Create references to cells A2:F2 in cells A61:F61.

Enter the function

`=Si6000b.xls!POLARSURFACEMICROSTRIP(A$61,B61,C$61,D$61,E$61,F$61)`

in cell G61. (Note the relative reference to cell B61.) Fill up to G56 and down to G66.

Create a step value of 0.10 in cell D53

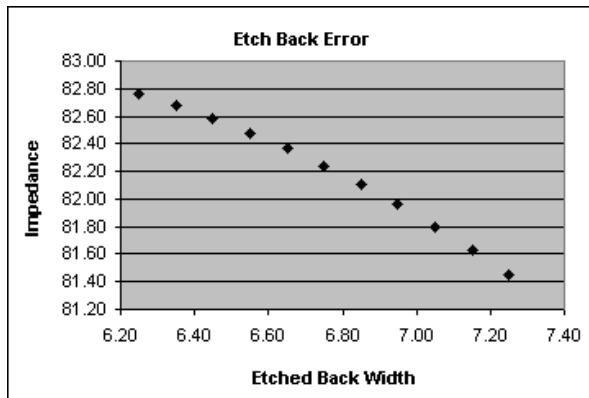
Enter the formula `=B61-$D$53` in cell B60 and fill up to B56.

Enter the formula **=B61+\$D\$53** in cell B62 and fill down to B56.

Audit the precedents and dependencies.

Select cell ranges W(var) B56:B66 and Z₀(W1) G56:G66 and chart.

The chart for an etch back error of 0.25 appears below.



Change the etch back factor cell value in cell D7 and recalculate to observe the change in impedance of a different trace side slope.

Appendix

For further relevant papers and application notes
visit the Polar Instruments web site,
www.polarinstruments.com.

Calculation of PCB Track Impedance

Andrew J Burkhardt, Christopher S Gregg and J Alan Staniforth

This paper discusses how high speed personal computers have allowed the use of accurate finite element numerical techniques to replace algebraic approximations

Introduction

The use of high-speed circuits requires PCB tracks to be designed with controlled (characteristic, odd-mode, or differential) impedances. Wadell^[i] is one of the most comprehensive sources of equations for evaluating these impedances. This source includes many configurations including stripline, surface microstrip, and their coplanar variants.

The IPC publication, IPC-2141^[ii], is another source of equations but has a smaller range of configurations, similar to those presented in IPC-D-317A.

However, for some configurations there are differences between the equations given in these publications. The authors believe that it is now opportune to examine the origin of the equations and to update the method of calculation for use with modern personal computers.

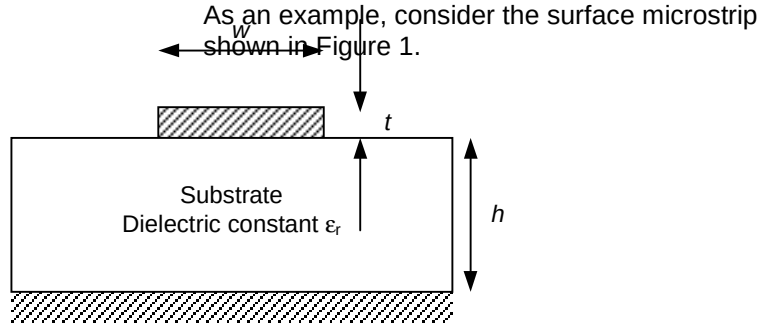


Figure 1 - Surface Microstrip

IPC-2141^[iii] gives the characteristic impedance as

$$Z_0 = \frac{87.0}{(\epsilon_r + 1.41)^{1/2}} \ln \left[\frac{5.98}{0.8w + t} \right] \quad (1)$$

Wadell^[i] gives

$$Z_0 = \frac{\eta_0}{2.0\sqrt{2.0\pi(\epsilon_r + 1)}^{1/2}} \ln \left[1.0 + \frac{4.0h}{w'} (A + B)^{1/2} \right] \quad (2)$$

$$\text{where } A = \frac{14.0 + 8.0/\epsilon_r}{11.0} \times \frac{4.0h}{w'} \quad (3a)$$

$$B = \left(A^2 + \frac{1.0 + 1.0/\epsilon_r}{2.0} \times \pi^{1/2} \right)^{1/2} \quad (3b)$$

$$\text{with } w' = w + \Delta w' \quad (3c)$$

The parameter w' is the equivalent width of a track of zero thickness due to a track of rectangular profile, width w and thickness t . Wadell^[i] gives an additional equation to

determine the incremental value $\Delta w'$. The parameter η_o , in equation (2), is the impedance of free-space (or vacuum), $376.7\Omega (\approx 120\pi)$. The quoted accuracy is 2% for any value of ϵ_r and w .

Table 1 shows the results of applying equations (1) and (2) to a popular surface microstrip constructed from 1oz copper track on $1/32$ inch substrate.

Table 1

Width w (μm)	Numerical Method $Z_o (\Omega)$	Equation (1)		Equation (2)	
		$Z_o (\Omega)$	% error	$Z_o (\Omega)$	% error
3300	30.09	21.08	-29.94	29.89	-0.66
1500	50.63	49.46	-2.31	50.50	-0.26
450	89.63	91.79	+2.41	89.89	+0.29

$t = 35\mu\text{m}$, $h = 794\mu\text{m}$, $\epsilon_r = 4.2$ (the calculation of the error assumes the numerical method is accurate: see Numerical Results)

Table 1 shows that equation (2) is well within the quoted accuracy. The accuracy of equation (1) varies widely, but this equation has the advantage of simplicity and is useful in illustrating the general changes to the value of Z_o as the width w and thickness t are varied.

The example demonstrated by Table 1, highlights the general problem with published equations: complicated equations are usually more accurate. Ranges over which the equations are accurate are also usually restricted to a limited range of parameters (e.g. w/h , t/h and ϵ_r).

Equation (2) is complicated, but with patience, can be evaluated using a programmable calculator or computer spreadsheet. However the complications increase greatly when two coupled

tracks are used to give a differential impedance. For coupled surface microstrip, Wadell^[i] gives 7 pages of equations to evaluate the impedance.

It is now a major exercise to evaluate the impedance using a calculator or spreadsheet.

Algebraic Equations

Single Track

For the stripline of Figure 2 with a symmetrically centred track of zero thickness, Cohn^[iii] has shown that the exact value of the characteristic impedance is

$$Z_0 = \frac{\eta_0}{4.0\sqrt{E_r}} \frac{K(k)}{K(k')} \quad (4)$$

$$\text{where } k = \operatorname{sech}\left(\frac{\pi w}{2.0h}\right) \quad (5a)$$

$$\text{and } k' = \tanh\left(\frac{\pi w}{2.0h}\right) \quad (5b)$$

K is the complete elliptic function of the first kind^[iv]. An equation for the evaluation of the ratio of the elliptic functions, accurate to 10^{-12} , has been given by Hilberg^[v], and also quoted by Wadell^[i].

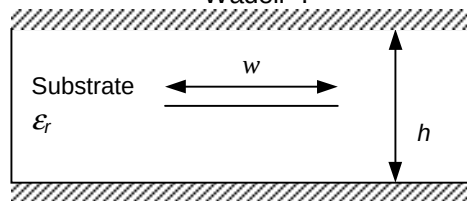


Figure 2 - Stripline: Centred Track

When the thickness is not zero, corrections have to be made which are approximate^[i]. These corrections are obtained from theoretical approximations or curve fitting the results of numerical calculations based on the fundamental electromagnetic field equations.

When the track is offset from the centre, the published equations become more complicated and the range of validity, for a given accuracy, is reduced.

Attempts have also been made to include the effects of differential etching on the track resulting in a track cross-section which is trapezoidal^[ii].

There is no closed-form equation like equation (4) for surface or embedded microstrip of any track thickness. Thus any equation used to calculate the impedance is approximate and demonstrated in Table 1.

Coupled Coplanar Tracks

Figure 3 shows two coupled coplanar centred stripline tracks.

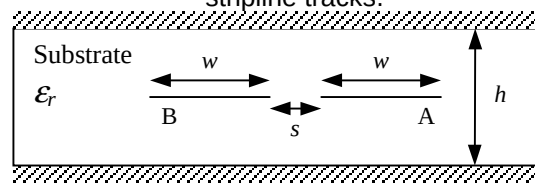


Figure 3 - Stripline: Coplanar Coupled Centred Tracks

All the impedance equations for coupled configurations refer to both even-mode impedance (Z_{0e}) and odd-mode impedance (Z_{0o}). These impedances are measured between the tracks and the ground plane. Z_{0e} occurs when tracks A and B are both at +V relative to the

ground plane, and Z_{0o} occurs when track A is at +V and track B is at -V. When a differential signal is applied between A and B, then a voltage exists between the tracks similar to the odd-mode configuration. The impedance presented to this signal is then the differential impedance,

$$Z_{diff} = 2 \times Z_{0o} \quad (6)$$

All published equations ^[i] give Z_{0o} . The differential impedance must then be obtained using equation (6).

For the zero thickness configuration of Figure 3, Cohn^[iii] gives the exact expression.

$$Z_{0o} = \frac{\eta_0}{4.0\sqrt{\epsilon_r}} \frac{K(k_0)}{K(k'_0)} \quad (7)$$

where $k_0 = (1 - k'^2_0)^{1/2} \quad (8a)$

and $k'_0 = \tanh\left[\frac{\pi w}{2.0h}\right] \coth\left[\frac{\pi(w+s)}{2.0h}\right] \quad (8b)$

As before K is the elliptic function of the first kind. There are no closed-form equations for coplanar coupled tracks.

Effect of Track Thickness

When the track thickness is not zero, approximations must be made to obtain algebraic equations similar to equations (4) and (7). Alternatively, equations, based on curve fitting of extensive numerical calculations, are used.

However, as the thickness increases the impedance decrease, as can be noted from equation (1).

Numerical Principles

For pulses on a uniform transmission system,^[i,vi] then

$$Z_0 \quad (\text{or } Z_{0o}) = \sqrt{\frac{L}{C}} \quad (9)$$

where L is the inductance and C the capacitance per unit length of line.

For a stripline, where the electric (and magnetic) fields are in a uniform substrate, dielectric constant ϵ_r , equation (9) becomes

$$Z_0 = \frac{\sqrt{\epsilon_r}}{cC} \quad (10)$$

where c is the velocity of light *in vacuo* (or free-space). The velocity of pulse travel along the transmission path is

$$v = \frac{c}{\sqrt{\epsilon_r}} \quad (11)$$

For a microstrip, the electric (and magnetic) fields are in air and the substrate, It can be shown that

$$Z_0 = \frac{1}{c\sqrt{CC_{air}}} \quad (12)$$

Where C_{air} is the capacitance of the same track configuration without substrate.

The effective dielectric constant is

$$\epsilon_{eff} = \frac{C}{C_{air}} \quad (13)$$

To find the impedance, the capacitance must be calculated.

This can be done by applying a voltage V to the tracks and calculating the total charge per unit length Q , from which

$$C = \frac{Q}{V} \quad (14)$$

However the surface charge on a track is not uniform. In fact it is very high at track corners. Therefore the total charge is difficult to calculate.

From electrostatic theory, it is known that a charge produces a voltage at a distance r from the charge.

hen a distribution of charge ρ (coulomb/unit width of track) gives a voltage

$$V = \int G \rho d\ell \quad (15)$$

where the integral is taken over the perimeter of the track cross-section, $d\ell$ is a small length, and G is the voltage due to a unit charge. It is also known as the Green's Function.

The value of G depends on the configuration (or environment). For instance, a point charge in a 2 dimensional dielectric space, without conductors gives

$$V = -\frac{\rho \ln(r)}{2\pi\epsilon_0\epsilon_r} \quad (16a)$$

so that
$$G = -\frac{\ln(r)}{2\pi\epsilon_0\epsilon_r} \quad (16b)$$

In equation (15), the voltage V is known, G is known for the particular configuration of tracks and substrate, but the charge ρ is unknown. Thus (15) is an integral equation which can be solved numerically by the Method of Moments (MoM)^[vii].

To proceed using MoM, the cross-section perimeter of the track is divided into short lengths with a node at each end. Charges are assigned to each node. The voltage at each node is calculated from all the nodal charges and the estimated charge variation between nodes. This leads to a set of simultaneous equations represented by the matrix equation

$$\mathbf{A} \mathbf{p} = \mathbf{V} \quad (17)$$

where $\mathbf{p}$ is a vector of nodal charges, and $\mathbf{V}$ is a vector of nodal voltages. $\mathbf{A}$ is a square matrix whose elements are calculated from integrals involving the Green's Function. The size of the matrices depends on the number of nodes. Equation (17) can be solved for the nodal charges $\mathbf{p}$ for given nodal voltages $\mathbf{V}$. The elements of $\mathbf{V}$ are usually +1 or -1 depending on the configuration.

The total charge Q can be obtained by a suitable summation of the nodal charges.

This general approach has been used by most authors to evaluate the various impedances. Most of the calculations were published 15 to 20 years ago, when the principal calculator was a main-frame computer. Hence the need for equations which could be used with the pocket calculators available at that time.

The present authors have revisited the basic numerical approach and have developed software^[viii] which readily calculates the controlled impedances using a desktop PC. The software runs quickly on a modern PC, and has been extended to also include the calculation of configurations not well represented in the literature.

This includes

- offset coupled stripline
- broadside coupled stripline
- embedded coupled microstrip

Thick tracks are normally to be expected which have a trapezoidal cross-section to allow for differential etching of the track.

Numerical Results

This section describes in more detail some of the numerical techniques and compares the results with the exact equations (4) and (7).

In all cases the Green's Function for the configurations was obtained using charge images in the ground planes. There are an infinite number of these images. In the case of stripline the sum of images converges to the result given by Sadiku^[ix]. Silvester^[x,xi] developed the image method for surface microstrip and has now been extended by the authors for embedded microstrip. In all cases the sum of images converges, but the result has to be obtained numerically.

The distribution of charge over an element between nodes is assumed to be linear. A numerical singularity occurs when the charge node j coincides with the voltage node i . Sadiku^[ix] indicated how this can be resolved. The evaluation of the elements A_{ij} consists of both numerical and analytic integration in the same manner as that used in Boundary Element techniques^[xii,xiii].

To avoid numerical inaccuracies at corners where there is a large concentration of charge, the length of an element at a corner is made very small. The other elements and nodes are then

distributed by the method described by Kobayashi^[xiv]. This means that wide strips require more nodes than narrow strips when the same small element is used.

The results presented were performed on a PC with an Intel Pentium Pro running at 233MHz using a compiled C-program.

Single Track Stripline

Figure 4 shows the variation of impedance with track width for the stripline of Figure 2.

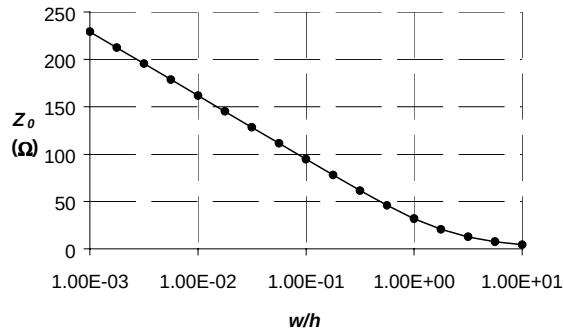


Figure 4 - Impedance for different relative width (Substrate $\epsilon_r = 4.2$)

Figure 5 shows the % error of the numerical calculation compared with the exact values given by equation (4). Two curves are shown for different small elements at the corner (i.e. ends of the track).

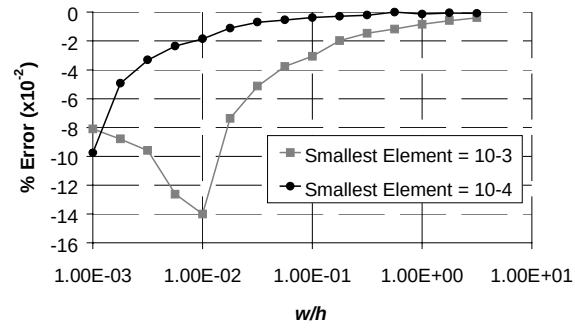


Figure 5 - Substrate $\epsilon_r = 4.2$

The above graph shows that good accuracy can be obtained over nearly four decades of the width/height ratio. The computer processing time was less than 0.5s for any of these values.

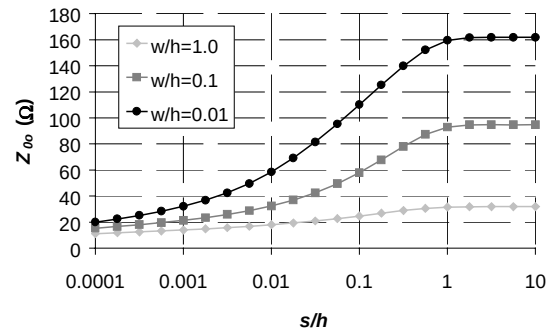


Figure 6 - Odd-mode impedance for different separations (s/h) and widths (w/h)

Coupled Coplanar Stripline

Figure 6 shows the variation of the odd-mode impedance for the stripline shown in Figure 3.

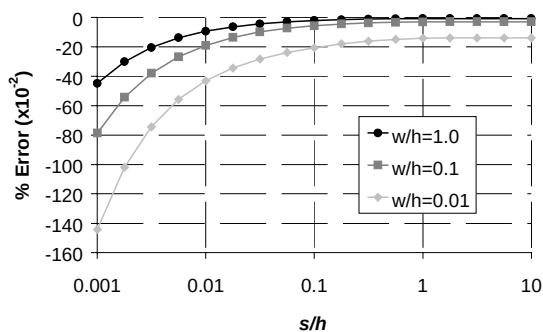


Figure 7 - % error $\mathcal{E}_r = 4.2$

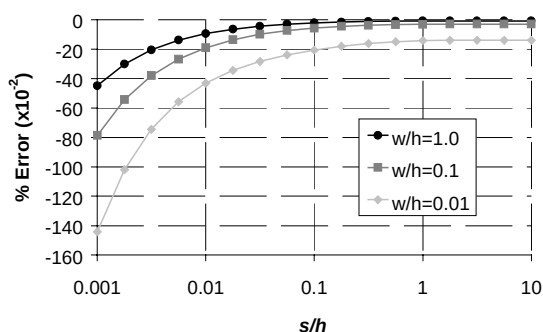


Figure 7 shows the % error of the numerical calculation compared with the exact values given by equation (7) using 10^{-3} as the smallest element. The maximum processing time was less than 0.5s. The maximum error can be reduced by

decreasing the smallest element. For a maximum error of $6.0 \times 10^{-2} \%$, a processing time of 5.1s is required.

The results presented in

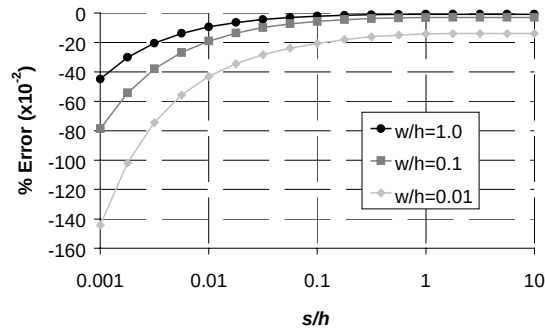


Figure 7 offer a very stringent test for the numerical method because of the sharp corners separated by s . In the odd-mode configuration this effect is enhanced even more because the tracks are of opposite polarity. This numerical validation is considered to be better then the results given by Bogatin et. al.^[xv] for a pair of 'round' tracks (i.e. a parallel wire transmission line) using finite element software. In this latter case there are no singularities at the corners. Li and Fujii^[xvi] state that the boundary element method (to which MoM is related) is more accurate for stripline and microstrip than the finite element method.

Surface Microstrip

As previously mentioned there are no closed-form algebraic equations which are exact. But the discussion in the previous sections shows that the software can be made accurate, especially for practical purposes. Table 1 shows calculations for

the configuration of Figure 1. Because the Green's Function involves a summation, and two capacitances C and C_{air} are required, processing times are now longer than those for stripline. The longest time was less than 4.5s for a width of 3300 μ m.

For coupled surface microstrip, two thick tracks of 3300 μ m requires a processing time of 5.1s. The separation does not affect the time.

Practical Results

In order to verify the practical performance of the field solving boundary element method, the authors commissioned production of a set of samples. During a six month period in 1998, over 1500 different printed circuit board tracks were manufactured.

This sample consisted of both stripline and microstrip differential structures in surface and embedded configurations. Two types of coupled structures were included; edge-coupled and broadside-coupled. The track dimensions ranged from 75 μ m to 1000 μ m in width, with differential separations of 1 track width to 4 track widths using base copper weights of ½oz, 1oz and 2oz. The resulting differential impedances ranged from 80 Ω to 200 Ω .

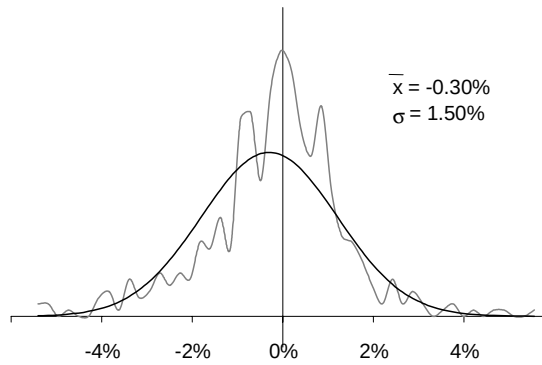


Figure 8 - Distribution of differences between predicted and measured values for stripline

Test samples were produced by three independent UK PCB manufacturers^[xvii] and the differential impedances were electrically measured by TDR at Polar Instruments using a CITS500s Controlled Impedance Test System.

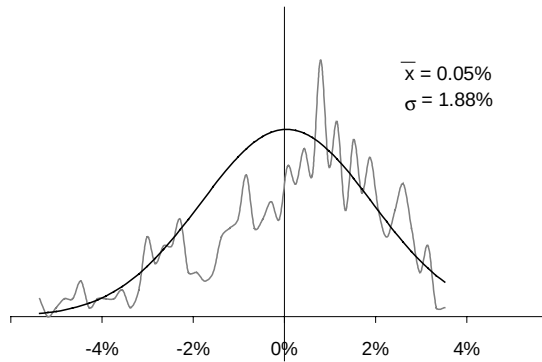


Figure 9 - Distribution of differences between predicted and measured values for embedded microstrip

After electrical measurement, the samples were returned to the manufacturers for microsection

analysis to determine the actual physical mechanical dimensions.

The calculated impedance was predicted from the mechanical microsection data and a derived value of relative permittivity, ϵ_r , of the FR-4 material. Results^[xviii] were analysed and comparisons of the electrically measured and the theoretically calculated results are presented in Figure 8 and Figure 9.

Discussion

Accuracy of the electrical measurements is estimated at 1% to 2%. This depends upon the impedance value and the quality of the interconnection between the test equipment and the test sample. Test samples were designed to be electrically balanced, but the manufacturing process will obviously not produce perfectly balanced traces.

Microsection dimensions have an estimated accuracy of 1%, however the model assumes symmetry and this will introduce a further small averaging error estimated at 1%. The total uncertainty in the experimental results is therefore estimated at 3% to 4%. Figure 8 and Figure 9 show mean deviations of less than 0.5% with standard deviations of less than 2%.

These practical results clearly show that the differences between the measured electrical results and the numerically calculated results are well within the estimated uncertainty of the measurement method.

Conclusion

The authors have shown that the early methods for calculating controlled impedance can now be used on desktop PCs. The accuracy is as good as, if not better than, the published algebraic equations. The processing times are less than 10 seconds which are acceptable in most cases.

Furthermore the number of configurations can be extended and trade cross-sectional profiles can be readily incorporated.

Calculation of Controlled Impedance

Article by Dr J. Alan Staniforth

2D Field solving in the Si6000

The *controlled* or *characteristic impedance* of a pair of conductors on a printed circuit board, is the ratio of voltage to current of the propagating pulse or step voltage. For the pulse or step to remain unchanged as it propagates, the impedance must be the same at each point on the propagation path. Reflections occur when the pulse encounters an impedance different from the controlled impedance.

For a single track above a ground plane, the controlled impedance is,

$$Z_o = \sqrt{\frac{L}{C}} \quad (1.1)$$

where L is the inductance and C is the capacitance per unit length of the track.

A pair of tracks of the same cross-section (i.e. balanced), above a ground plane, have an impedance,

$$Z_{oo} = \sqrt{\frac{L_{oo}}{C_{oo}}} \quad (1.2)$$

to ground when they are excited by an equal and opposite voltage to the ground voltage. Z_{oo} is also known as the *odd mode impedance*. L_{oo} and C_{oo} are, respectively, the odd mode inductance and capacitance per unit length.

The impedance between the tracks is the *differential impedance*,

$$Z_{diff} = 2Z_{oo} \quad (1.3)$$

It is only possible to derive algebraic equations for the various inductances and capacitances for a very limited number of track configurations, usually when the track thickness is zero.

Modern tracks may be relatively thick, and not rectangular, because of the effect of the etching process. In these circumstances, the value of the inductance and capacitance, and hence impedance, must be obtained directly from electromagnetic field calculations.

For striplines which have a uniform substrate, dielectric constant ϵ_r , (1.1) can be written

$$Z_o = \frac{\sqrt{\epsilon_r}}{cC} \quad (1.4)$$

where c is the velocity of light.

For both surface and embedded microstrip lines, (1.1) can be written

$$Z_o = \frac{1}{c\sqrt{CC_{air}}} \quad (1.5)$$

where C_{air} is the capacitance of the same configuration with air as the substrate.

The effective dielectric constant is

$$\epsilon_{eff} = \frac{C}{C_{air}} \quad (1.6)$$

Similar equations for Z_{oo} also apply.

Calculation of Capacitance

The equations in section 1 assume that the electric fields can be obtained from electrostatic field theory. This is acceptable when the track dimensions are small relative to the wavelength of the highest frequency component in a pulse or step.

When the track is at voltage V , relative to the ground, then

$$C = \frac{Q}{V} \quad (2.1)$$

where Q is the charge per unit length of track. In (2.1), C can be any of the capacitances in the equations of section 1. The charge density, q over the track cross-section, is not uniform: there will be a much greater density at track corners and edges. At the pulse rates and frequencies involved, the charge density resides on the track surface. If the distribution of q is known, then

$$C = \sum_{\substack{\text{over} \\ \text{track} \\ \text{surface}}} q \quad (2.2)$$

Many methods are available to estimate q . A popular method is the finite element method. In this method, the whole of the transverse plane containing the track is divided into a mesh of triangles of different sizes. The relation between the unknown voltages at the nodes of the triangles, is based on the minimum electrostatic energy, and is known. Some nodes are on the surface of the track. At these nodes the voltage is known. Then by solving a matrix equation, all the unknown voltages and track surface charge densities are found. The difficulty with this method is that the voltages in the space around the track are unnecessarily calculated. Also it is difficult to ensure that the high charge density at the track corners is adequately modelled since most field solvers using this method, make several passes through the program each time with a finer mesh. The mesh sizes are determined within the software. The program terminates when there is very little change between adjacent passes.

Method of Moments

The method used in CITS25 and SI6000 to estimate q , is the method of moments.

This method is based on the direct relation between voltage and charge in an electrostatic field. The voltage V at a distance r from a charge q in a two dimensional region is

$$V = -\frac{q \ln(r)}{2\pi\epsilon_o\epsilon_r} \quad (2.3)$$

where ϵ_o is the absolute permittivity and ϵ_r is the dielectric constant of the region. (2.3) can be written as

$$V = -qG \quad (2.4)$$

where G is known as the Green's function. For a simple two dimensional region, G can be found by comparing (2.4) with (2.3). When q is a continuous distribution of charge, then (2.4) is modified to read

$$V = - \int_{\substack{\text{charge} \\ \text{surface}}} qGdl \quad (2.5)$$

To apply the method of moments, the track is assumed to have a known voltage, V . The track surface is assigned nodes which have a predetermined very non-linear distribution. These nodes are very close together at the corners. An unknown charge, q_j , is assigned to the nodes.

The charge distribution between nodes is assumed to be linear. Using (2.5) with q replaced by q_j , the known voltage, V_i at the nodes, is calculated from the charge at all the nodes. This is known as point matching. The integration in (2.5) is obtained numerically using Gauss integration. This process leads to the matrix equation

$$\mathbf{A}\mathbf{q} = \mathbf{V} \quad (2.6)$$

where $\mathbf{A}$ is a known square matrix derived from the application of (2.5) with the appropriate Green's function, $\mathbf{q}$ is a vector of the unknown nodal charges, and $\mathbf{V}$ is a vector of the known nodal voltages. The solution of (2.6) yields the nodal charges. Using the nodal charges, the track charge Q is obtained by summation assuming the linear charge distribution between nodes, from which the capacitance and impedance is

calculated. In the microstrip case, there are two calculations required; the first with the substrate, and the second with the substrate replaced by air.

The above is an outline of the calculation. The calculation uses the appropriate Green's function for the track configuration. The steps used to obtain the Green's function for the track configurations, is described in the next section

The Green's Functions.

A simple two dimensional Green's function is outlined in (2.4). However this value cannot be used directly in the calculation, because there are ground planes and substrate surfaces present.

In a stripline there are two ground planes. A charge q lies between them in a uniform substrate. Fig 1 shows the general arrangement.

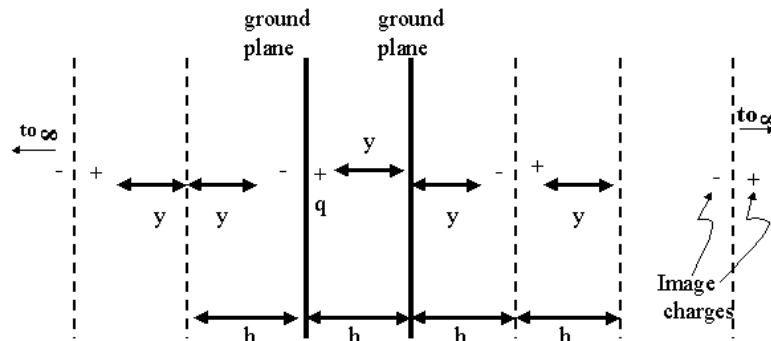


Fig. 1. Charge between Ground Planes

The charge images are of equal and opposite sign and extend to infinity in each direction. The ground planes can be notionally removed, and

the charge images used instead. The Green's function is obtained by summing, algebraically, the individual effect of an infinite number of charges, with q equal to 1.

Fig. 2 shows the images for a charge q above the substrate of a surface microstrip.

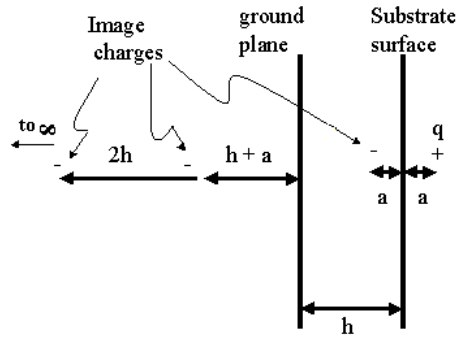


Fig. 2. Charge above Substrate Surface

There are image charges produced by the multiple reflections between the substrate surface and the ground plane. The images are of the opposite sign to that of q and decrease in amplitude as the distance is increased. The ground plane and substrate surface are replaced by the infinite number of image charges. The Green's function is the sum of the effect of these images when q equals 1. The sum cannot be obtained analytically.

Fortunately the infinite sum quickly converges as the number of terms increases. The sum is readily calculated numerically to high degree of accuracy.

The Green's function for embedded microstrip, is derived in a similar manner to that for surface microstrip, except that the pattern of images is similar to those for stripline.

The Program

The program used to calculate the impedances, assumes that, in manufacture, the tracks are etched to a trapezoidal shape. For single tracks, the whole width of the track is used. For pairs of tracks, symmetry is assumed. Only nodes on one track need then be used. The odd mode impedance, Z_{oo} , is first calculated.

The differential impedance is then obtained using (1.3). Within the program, all dimensions are normalised to the substrate thickness. This means that any consistent set of dimensions can be used to define the track configuration. During the development of the software, various combinations of the number of nodes and their non-linear distribution, were examined. The impedances calculated were compared with the impedances for those few cases where there is an exact result. This comparison shows that the impedances calculated by Si6000 are accurate to at least 3 significant figures in the range 20 – 150Ω.

Terms used in this manual

AC	Alternating Current
CMOS	Complimentary Metal Oxide Silicon
DC	Direct Current
ECL	Emitter Coupled Logic
EMI	Electromagnetic Interference
FR-4	Epoxy Glass Dielectric Material
TDR	Time domain Reflectometry
TTL	Transistor-Transistor Logic
Z_0	Characteristic Line Impedance
Z_0'	Characteristic Line Impedance (Loaded)
E_r	Relative Permittivity (homogeneous dielectric materials)
E_r'	Effective Relative Permittivity (non-homogeneous dielectric materials)

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^{xvii} The authors wish to acknowledge the assistance of Kemitron
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^{xviii} Surface microstrip results were yet to be completed at the submission
date for this paper.